Title: Another Way of Factoring

Brief Overview:

Students will find factors for quadratic equations with a leading coefficient of one. The students will then graph these equations using a graphing calculator and analyze the factors and the graphs of the equations in order to determine a relationship between the two. This relationship will then be used to help factor more difficult polynomial equations. Students will also explore a real-life application in physics.

Links to Standards:

- **Mathematics as Problem Solving**
  The students will demonstrate the ability to gather information from a graphical representation of an equation.

- **Mathematics as Communication**
  In their journals, students will explore the differences and similarities between factoring polynomial equations graphically and analytically.

- **Mathematics as Reasoning**
  The students will explain the connection between the graphical data and the factored form of the equation.

- **Mathematical Connections**
  The students will explore the use of quadratics in physics.

- **Algebra**
  The students will be able to use an equation to represent a given situation, then graph the equation to attain results or solutions that satisfy the given situation.

- **Functions**
  The students will use the graphs of quadratic functions with a leading coefficient of one, to analyze the relationship between zeros and factors of polynomials.

- **Technology**
  The students will use graphing calculators to graph and analyze polynomial functions. This lesson is designed for the TI-83 calculator, and adjustments may need to made for other calculators.

Links to Maryland High School Mathematics Core Learning Goals

- **1.1.1:** The student will recognize, describe, and extend patterns and functional relationships that are expressed numerically, algebraically, and geometrically.

- **1.1.2:** The student will represent patterns and functional relationships in a table, as a graph, and/or by mathematical expression.

- **1.1.4:** The student will describe the graph of a non-linear function in terms of the basic concepts of maxima and minima, roots, limits, rate of change, and continuity.

- **1.2.4:** The student will describe how the graphical model of a non-linear function represents a given problem and will estimate the solution.

Grade/Level:

8 - 12 This lesson can be used for students just learning to factor; students having problems factoring; or as a review of factoring.

Duration/Length

Four 45-minute periods or two 90-minute periods

Prerequisite Knowledge:

Students should understand or be able to do the following:

- Factoring $ax^2 + bx + c$, where $a = 1$ over the rational numbers.
- Use inverse operations on equations.
- The zero-product rule. (If $ab = 0$, then $a = 0$ or $b = 0$)
- Use the Graph and Calculate features of the graphing calculator.
- The relationship between points on the coordinate plane and an equation.

Objectives:

Students will be able to:

- factor quadratics of the form $ax^2 + bx + c$ (over the rational numbers).
- solve quadratic equations graphically.
- use a given set of solutions to find the original quadratic equations.

Materials/Resources/Printed Materials:

Teacher:

- TI-83 overhead kit
- Teacher notes
- Hint sheet for students (Hint sheet should be left in a central location, accessible to students.)

Students (groups of two):

- TI-83 calculator
- Activity sheets

Development/Procedures:

- Start the lesson by reviewing factoring quadratics with a leading coefficient of one, and how the zero product rule is used to solve equations.

- Review graphing techniques on calculator. Students need to be able to graph, find the zero of a graph, and adjust the viewing window. Students also need to be able to convert decimals into fractions.

- Students work on "Is There an Easier Way to Factor?--Part One." Teacher should observe groups, and assist students as needed. The hint sheet can be put on the teacher's desk for those students who would like some assistance, but do want to ask the teacher.

- After students have finished Part One, discuss their findings as a class. It is important that all students see the connection between the solutions and the factors of a polynomial.

- Students work on "Is There an Easier Way to Factor?--Part Two." Teacher should observe groups, and assist students as needed.

- After students have finished Part Two, discuss their findings as a class. Students should create a Problem to Pass.

- Students work on application sheet. Teacher should observe groups, and offer assistance as needed.

- Finish the lesson by discussing concepts learned. Assign journal entry, and give quiz.

Performance Assessment:

- Teacher will informally assess students by observing students as they work on activity sheets.

- Students will create a Problem to Pass. Each group writes a problem, similar to the ones on the worksheets, on an index card with a complete answer and explanation on the back. These cards are then traded among groups. Each group then answers the question passed to them. The class can discuss the problems created.

- The Applications worksheet will be a formal assessment.

- The students will take a quiz at the end of the lesson.

- The students will write in their math journals describing the relationship between the solutions and the factors of a polynomial. Students should also discuss the different methods they know for factoring a polynomial.

Extension/Follow Up:

- Students could explore other characteristics of a quadratic function; such as, the vertex or how the coefficients affect the shape and position of the parabola.

- Students could explore higher degree polynomials. Students could be introduced to the theorems that describe the behavior of polynomials; such as, Rational Root Theorem, Descartes' Rule, or the Intermediate Value Theorem.

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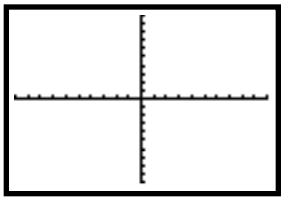
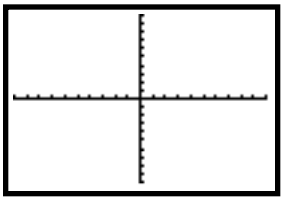
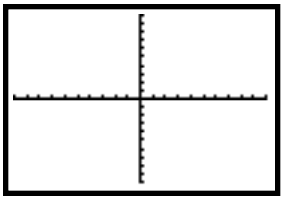
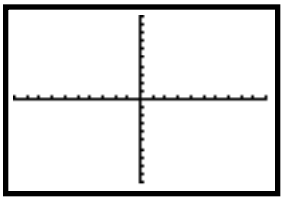
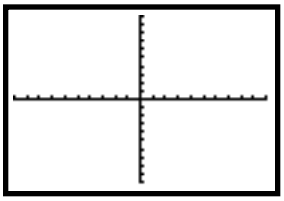
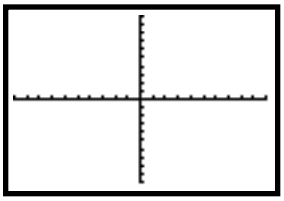
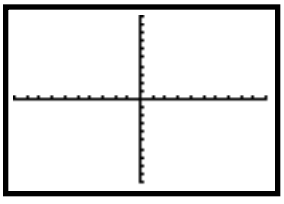
Is There an Easier Way to Factor?--Part One

Directions: Factor and solve each quadratic. Use your calculator to graph each one. Be sure to write solutions as fractions.

Quadratic	Factored Form	Solutions	Graph
$x^2 + 10x + 9 = 0$			
$x^2 - 5x + 6 = 0$			
$x^2 - 5x - 36 = 0$			
$x^2 + 14x + 49 = 0$			
$x^2 + 20x + 100 = 0$			
$x^2 + 4 = 0$			
$x^2 + 4x + 2 = 0$			

Is There an Easier Way to Factor?--Part One

Directions: After you have factored, solved, and graphed all the quadratics, answer the following questions. Be prepared to discuss what you have learned.

1) Describe the relationship between the solutions and the factors.

2) When the graph just touches the x-axis, how many solutions does this represent? How do you write the factored form?

3) How can you tell from the graph if a polynomial is prime?

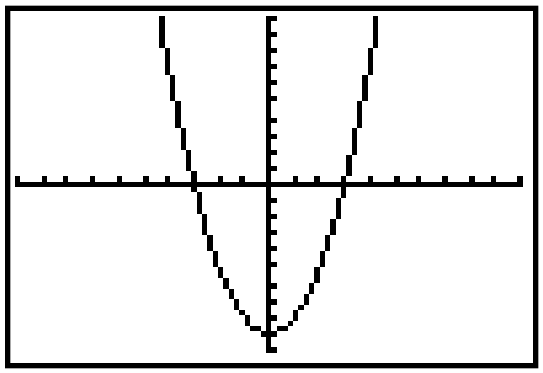
4) Is every polynomial factorable, if its graph crosses/touches the x-axis?

Directions: Use each set of solutions to find the original quadratic equation. Your equation should have integral coefficients. Graph your equations to check your work.

Solutions	Original Equation
$x = 3$ or $x = -2$	
$x = -3/4$ or $x = -5/2$	
$x = 4$	
$x = -0.5$ or $x = 1.75$	

Is There an Easier Way to Factor?--Part Two

Using the following graphs, write the factored form of the equation whose graph is shown.



x scale = 1, y scale = 1

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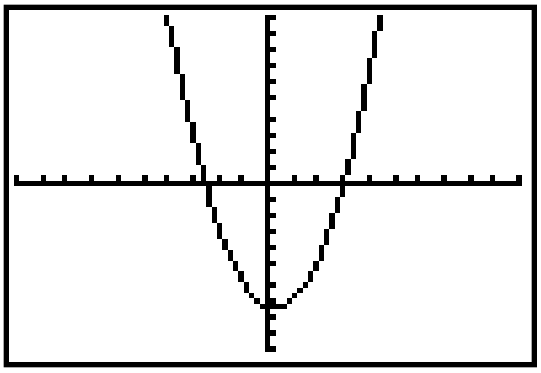

x scale = 1, y scale = 2

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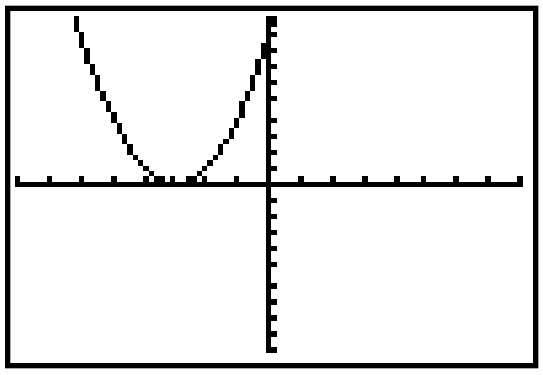
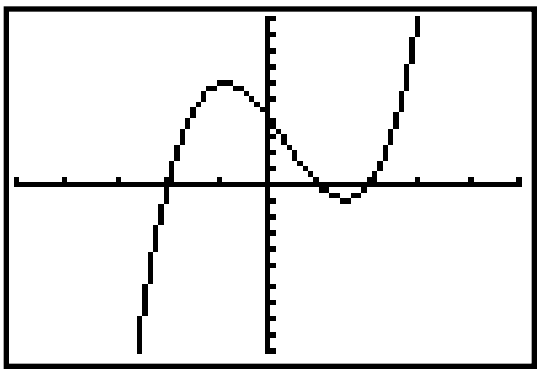


x scale = 1, y scale = 1

_____



x scale = 1, y scale = 1

_____

Is There an Easier Way to Factor?--Part Two

Factor the following polynomials. (Be sure to factor completely.)

$16x^2 - 8x - 15$ _____

$6x^2 - x - 5$ _____

$2x^2 + 9x - 11$ _____

$22x^2 + 47x + 6$ _____

$12x^2 - 7x - 10$ _____

$4x^2 - 25$ _____

1) Factor $8x^2 - 24x + 16$. _____

Did you get $8(x - 2)(x - 1)$? If not, how does your answer differ?

When factoring, what should you always do first?

What is the completely factored form of $8x^2 - 24x + 16$?

2) Explain how you would factor $7x^2 - 19xy + 10y^2$ using your calculator.

Application: Projectile Motion in Physics

Examples of projectile motion occur often in physics. When a rocket is launched it is necessary to know how long it will take to land, how high it will travel, and how long it takes to reach certain heights or distances.

It was Galileo Galilei (1564-1642) who first accurately described projectile motion. In the 1600's, Galileo proved that when a frictionless object is projected up from the ground, it takes the same amount of time to go up as it takes to come back down. His work laid the foundation for the studies of Sir Isaac Newton who formulated the laws of gravity and motion. The application of these formulas led to successful space travel.

The path that a projected object travels is a parabola, represented by the function:

$$\mathbf{H(t) = -\frac{1}{2}gt^2 + vt}$$

where H(t) = height
t = time
g = force of gravity (32 ft/sec^2 on Earth)
v = initial speed or initial velocity

*set graphing range on calculators so that $0 \leq x \leq 8$ with an x scale of 0.5 and $-20 \leq y \leq 200$ with y scale of 10

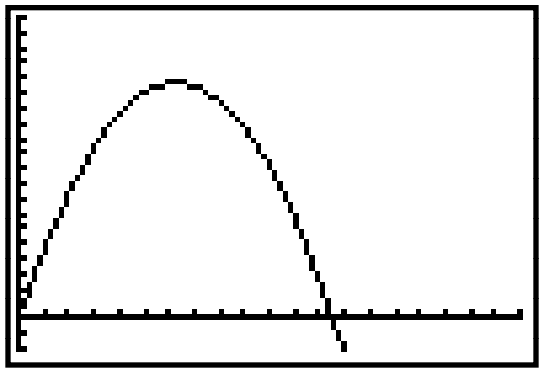
EXAMPLE A rocket is launched from Earth with an initial speed of 100 ft/sec. Graph the function that models the path of the rocket.

$g = 32 \qquad v = 100$

$$H(t) = -\frac{1}{2}gt^2 + vt$$

$$= -\frac{1}{2}(32)t^2 + (100)t$$

$$H(t) = -16t^2 + 100t$$



x scale = 0.5, y scale = 10

EXERCISES

Use the previous graph to answer Exercises 1 to 4.

1. What real-world occurrence do the x- and y-axes represent in this graph?

2. How long it will take before the rocket lands?

3. What is the maximum height the rocket will reach?

4. How long does it take for rocket to reach its maximum height?

5. A rocket is propelled upward with an initial speed of 120 ft/sec.

   a. How high will this rocket go?

   b. How long will it take the rocket to land?

   c. At what time will the rocket be at 100 ft of height?

PROJECT

Investigate the effects of the forces of gravity for various planets. Using the force of gravity for three other planets, answer the following questions for each planet. Assume the rocket is launched with an initial speed of 100 ft/sec.

   a. How high will the rocket go?

   b. How long before the rocket hits the ground?

   c. Suppose you are launching the same rocket on each of your three planets. You want each rocket to reach the same maximum height as it did on Earth. Determine the initial speed needed on each planet.

Teacher Notes

Is There an Easier Way to Factor?--Part One

Before handing worksheets to students:

- Review basic factoring techniques with students. The students need to be proficient at factoring quadratics of the form $ax^2 + bx + c$, where $a = 1$.
- Review calculator techniques with students. Students need to be able to graph equations. Students will need to find the zeros of an equation. Review with students how to convert decimals to fractions. Students need to be able to determine whether a number is rational or not.
- Example: Solve $2x + 5 = 0$, graphically.

While students work:

- Observe students as they work. Check students' first few problem to be sure they are filling in all parts correctly. It would be best if students do each problem completely before they attempt the next. Some students may need some "guidance" in stating the relationship between the solutions and the factors of a polynomial.
- Most students will need help understanding how you can tell from the graph whether a polynomial is prime. Stress that when the solutions are non-repeating decimals or if there are no solutions, the polynomial is prime (over the rational numbers). Remind students that non-repeating decimals will not convert to fractions, since they are irrational.

Is There an Easier Way to Factor?--Part Two

Before handing worksheets to students:

- Review concepts from part one. Students need to understand the relationship between the solutions and the factors. Be sure to clarify any questions from the first part.

While students work:

- Assist students in reading graphs and writing equations.
- Stress to students that the calculator will not show the greatest common numerical factor. Students should be told to find the gcf first when starting any problem, and include it in the factored form.
- Show students how to use calculator for two variable equations. Students can leave out the second variable while using the calculator, and then put the variable in the answer. Students need to understand factoring patterns in order to use the calculator for this type of problem.

Application: Projectile Motion in Physics

- Students who choose to do the project will need to find the forces of gravity for other planets. This information can be found by either traditional means or on the Internet.

Extension Problem:

The marketing department at Rock-n-Rollerblades found that, on the average, they sell 600 pairs of rollerblades monthly at a price of $100 each. The department also knows that for each $5 reduction in price, there will be an extra 50 rollerblades sold each month.

How low can Rock-n-Rollerblades drop their prices before their monthly sales income drops below the current monthly sales income?

What is the best price for the company to charge to reach maximum monthly sales income?

Teaching points for extension:

- Have students brainstorm session in small groups, and then follow with a discussion by whole class.
- Demonstrate how factors relate to real world problems.
- Use the graphing calculator to help illustrate concepts.

Steps:

1. Brainstorm ideas
2. Set up quadratic expression and graph
   price = 100 - 5x sales = 600 + 50x
   income = price x sales
   = (100 - 5x) (600 + 50x)
3. Graph equation to view curve. Students should attempt to set good parameters to view window.
4. Use the original profits of $60,000 as baseline by overlapping graph
   Y = 60,000.
   Example viewing window Xmin 50,000 Xmax 70,000 Xscale 1000
   Ymin 0 Ymax 10 Yscale 1
5. Use graph to answer the two questions.
   Remember x = number of 5 dollar discounts.

Hint Sheet

Answer for number 1 (Is There an Easier Way to Factor?--Part One)

$x^2 + 10x + 9 = 0$ $(x + 9)(x + 1)$ $x = -9$ or -1

Prime polynomials

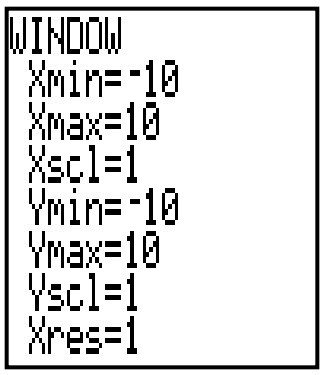
Can you write the solutions as a fraction?

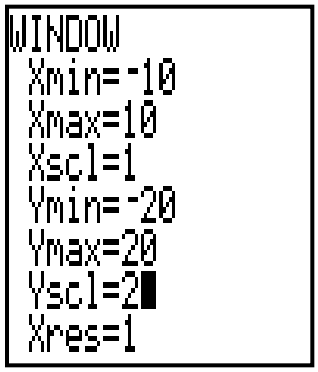
Are there any points where the graph crosses or touches the x-axis?

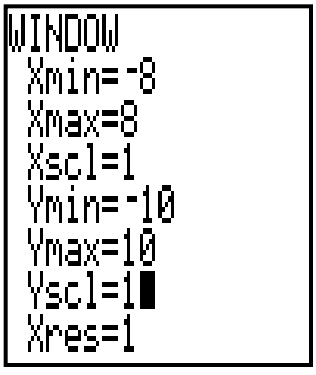
Finding equations if you know the answer.

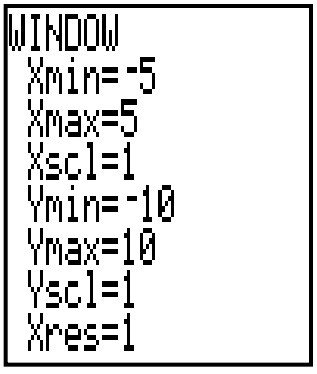
For each solution, try to solve and have one side equal to zero.

Viewing windows for the graphs Part Two.









Quiz

You may use your calculator for this quiz. Be sure to factor completely. Use fractions, not decimals!

Factor the following.

1) $x^2 + 4x - 21$

2) $2x^2 + 5x + 3$

3) $4x^2 + 4x + 1$

Find the original quadratic equation for the following solutions. Your equation should have integral coefficients.

4) $x = -3$ or $x = 2$

5) $x = 3/2$ or $x = -1$

Challenge

Factor the following $x^3 + 9x^2 + 27x + 27$ completely. Describe how you found the solutions.